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Near Infrared Photometry of Late-Type Stars

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Near infrared photometry of late-type stars

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A multi-band photoelectric photometer for observations in the ultraviolet, blue and visible and the infrared bands W (1·06 μm), X (1·13 μm), Y (1·63 μm) and Z (2·21 μm) has been constructed and applied to both stellar and planetary observations.

The results of photometry obtained for 61 stars are presented. The observations at $\lambda = 1{\cdot}63\,\mu$m are shown to exhibit an excess flux due to a minimum in the H^- opacity in accordance with the predictions of model atmosphere studies. Bolometric corrections are derived for stars of late spectral type from integration of the observed absolute spectral irradiance curve. A simple photometric method for the measurement of stellar diameters is proposed based upon the absolute irradiance observed at 2·21 μm and the 2·21 μm flux at the surface of the star calculated from model atmospheres. Angular diameters derived by this technique are consistent with interferometric results; and, when combined with the bolometric corrections, effective temperatures are found.

The photometer

During the last few years techniques for observation at infrared wavelengths have greatly improved. In particular the development of multi-layered interference filters for the infrared region now permits one to isolate narrow well-defined spectral bands centred on almost any desired wavelength. Of equal importance has been the rapid growth in solid-state electronics technology and components. These developments result in a great simplification of the photometer electronics and recording system and contribute substantially to the stability and over-all accuracy of the observations.

The photometer employed here is a dual-channel instrument using a 1P21 photomultiplier tube for observations in the ultraviolet, blue and visible and a lead sulphide detector for the near infrared. Both detectors are cooled by a slurry of dry ice in freon. Filters of Corning and Schott glasses are used for ultraviolet, blue and visible, while multilayed interference filters are used in the infrared.

An important feature of the photometer is the conical blackbody cavity mounted in the Cassegrain adaptor section. This cavity may be operated at any closely regulated temperature in the range from ambient to 500 °K. All the infrared observations were referenced to this blackbody source, thus making possible a direct determination of the absolute value of the radiation flux incident at the focal plane of the telescope.

Table 1 summarizes the pertinent features of the photometer and defines the infrared magnitudes W, X, Y and Z. The column headings are the 'effective wavelength', λ_e, of the normalized filter-detector response; its equivalent width, $\Delta\lambda$; and the absolute response in terms of the spectral irradiance $H(\lambda)$ for a star of zero magnitude. Details of the photometer design and performance, reduction of the observations, and absolute calibration of the spectral bands have been given by Walker (1966).

TABLE 1. CHARACTERISTICS OF THE PHOTOMETER, AND CALIBRATION IN TERMS OF THE SPECTRAL IRRADIANCE $H(\lambda)$ FOR A STAR OF ZERO MAGNITUDE

band	λ_e (μm)	$\Delta\lambda$ (μm)	$H(\lambda)$ (W cm^{-2} μm^{-1})
U	0·36	0·047	$4{\cdot}35 \times 10^{-12}$
B	0·44	0·093	$7{\cdot}17 \times 10^{-12}$
V	0·56	0·064	$3{\cdot}72 \times 10^{-12}$
W	1·06	0·077	$6{\cdot}47 \times 10^{-13}$
X	1·13	0·114	$5{\cdot}32 \times 10^{-13}$
Y	1·63	0·173	$1{\cdot}55 \times 10^{-13}$
Z	2·21	0·271	$5{\cdot}11 \times 10^{-14}$

THE OBSERVATIONS

Observing sessions, each extending from 1 to 3 weeks in duration, were carried out in October, November and December 1964, and in March and April 1965, with the photometer at the straight Cassegrain focus of the 61 in. telescope at Agassiz Station, Harvard, Massachusetts. A total of 15 nights, or partial nights, of good photometric quality occurred during the above sessions. The results discussed here are based upon the photometry obtained on those nights.

The magnitudes and colours observed for 61 stars are summarized in table 2. The observations are principally of stars of late spectral type, only ten with spectra earlier than the Sun being included. Entries in the table need little elaboration. The spectral types are on the revised MKK system where possible; however, many late type stars have not been reclassified. The colours and magnitudes tabulated are corrected to the top of the atmosphere and, for the ultraviolet, blue and visible are reduced to the system of Johnson & Morgan (1953). The final column, which is headed 'number', gives the number of occasions on which the star was observed. In most cases this refers to different nights; however, the stars α Aur, α Tau, β Cnc, and α Boo were often observed both early and late in the night as a check on the constancy of the atmosphere and to serve as a means of intercomparison of observations. All stars were observed on at least two occasions with the exception of κ Cet, 11 Leo Minor and HR 36395. A minimum of two observations was made in each colour on each night and the results were averaged.

Table 3 presents the observations reduced to absolute spectral irradiance observed at the effective wavelength of the system response. Although the data are tabulated to three figures, the third is not considered significant. The values listed for 0·36, 0·44 and 0·56 μm were calculated from the calibrations given in table 1 and the observed magnitudes and colours, while the infrared values result from a direct reference to the blackbody radiation source. In actual operation at the telescope the blackbody source was operated at a carefully controlled temperature, $T = 402$ °K, and considered the fundamental reference. Any deviations from the mean recorder deflexion observed on this reference were attributed to short-term variations in the photometer absolute response. The measured magnitudes were corrected for these variations in the data reduction process. For ultraviolet, blue and visible a check on the constancy of the zero point of the photometry was made by frequent observation of a tritium-activated phosphor mounted within the photometer.

Calibration of the visible magnitude in terms of absolute radiometric quantities has been discussed by Walker (1964) from consideration of the solar irradiance as given by

Table 2. Observed magnitudes and colours

name	sp. type	U–V	B–V	V	V–W	V–X	V–Y	V–Z	b.c.	number
β And	M0 III	3·52	1·57	2·06	2·36	2·50	3·43	3·54	−1·40	2
γ And	K3 II+A	2·73	1·34	2·32	1·82	1·91	2·70	2·83	−0·86	3
λ And	G8 III–IV	1·67	1·00	3·74	1·25	1·39	2·05	2·14	−0·41	2
R And	S4	3·43	1·63	6·68	3·08	3·27	4·35	4·62	−2·21	2
β Aql	G8 IV	1·24	0·79	3·72	1·09	1·19	1·76	1·76	−0·27	3
λ Aqr	M0	2·66	1·62	3·91	2·92	2·95	4·16	4·28	−1·98	2
α Ari	K2 III	2·22	1·15	2·07	1·39	1·49	2·21	2·39	−0·52	4
β Ari	A5 V	0·23	0·13	2·65	0·00	−0·01	−0·01	−0·04	—	2
α Aur	G8 III+F	1·24	0·80	0·07	0·91	0·98	1·41	1·46	−0·12	15
α Boo	K2 IIIp	2·48	1·23	−0·07	1·47	1·62	2·38	2·49	−0·60	9
η Boo	G0 IV	0·79	0·58	2·67	0·54	0·63	0·87	0·86	0·07	4
α C Ma	A1 V	−0·04	0·00	−1·46	−0·08	−0·13	−0·16	−0·16	—	2
α C Mi	F5 IV–V	0·47	0·43	0·37	0·44	0·52	0·69	0·80	—	2
α Cet	M2 III	3·67	1·64	2·53	2·62	2·75	3·79	3·99	−1·70	2
κ Cet	G5 V	0·91	0·68	4·83	0·42	0·65	0·74	0·82	—	1
o Cet	M7e	2·18	1·55	6·34	6·20	7·08	8·19	8·70	−5·90	2
γ Cet	G8 Vp	0·94	0·73	3·51	0·91	1·00	1·56	1·63	−0·18	5
β Cnc	K4 III	3·22	1·48	3·55	1·94	2·10	3·00	3·12	−1·03	30
RS Cnc	M6	2·97	1·68	5·38	5·14	5·36	6·48	6·85	−4·30	2
β Com	G0 V	0·64	0·58	4·24	0·29	0·49	0·81	0·86	—	2
Y CVn	N3	—	2·87	5·35	3·62	3·54	5·14	5·66	−2·93	2
ε Cyg	K0 III	1·89	1·03	2·45	1·23	1·32	2·03	2·11	−0·39	6
γ Eri	M0 III	3·36	1·49	2·90	2·41	2·50	3·57	3·71	−1·49	2
ε Eri	K2 V	1·51	0·92	3·75	1·10	1·13	1·70	1·87	−0·26	2
16 Eri	gM3	3·15	1·54	3·45	3·19	3·30	4·35	4·68	−2·26	2
β Gem	K0 III	1·85	1·00	1·16	1·13	1·23	1·78	1·86	−0·28	18
η Gem	M3 III	3·17	1·60	3·19	2·93	3·02	4·02	4·28	−1·96	2
κ Gem	G8 III	1·63	0·93	3·57	1·06	1·16	1·71	1·76	−0·24	8
μ Gem	M3 III	3·76	1·71	2·83	2·92	3·12	4·11	4·45	−2·04	2
51 Gem	M3	3·48	1·68	5·04	3·46	3·72	4·80	4·98	−2·60	2
R Lep	N6e	—	—	8·31	5·19	5·42	7·25	8·27	−5·13	3
α Lyr	A0 V	−0·01	0·00	0·04	−0·19	−0·27	−0·25	−0·24	—	5
δ² Lyr	M4 II	3·33	1·67	4·31	3·80	3·87	5·16	5·29	−2·90	2
11 L Mi	G8 IV–V	1·21	0·97	5·44	0·81	0·94	1·42	1·15	−0·05	1
α Ori	M2 Iab	3·93	1·87	0·69	3·11	3·16	4·23	4·56	−2·16	2
BL Ori	NB	6·21	2·40	6·32	3·53	3·57	4·98	5·24	−2·72	2
o Ori	gM4	3·85	1·80	4·68	3·29	3·44	4·57	4·87	−2·42	2
β Peg	M2 II–III	3·62	1·67	2·54	2·94	3·10	4·21	4·46	−2·07	2
γ Peg	B2 IV	−1·10	−0·23	2·83	−0·62	−0·91	−0·97	−0·98	—	2
η Peg	G2 II–III	1·42	0·86	2·93	0·96	1·13	1·73	1·78	−0·22	2
χ Peg	M2 III	3·50	1·58	4·78	2·46	2·60	3·54	3·78	−1·52	4
55 Peg	M2 III	3·46	1·56	4·52	2·34	2·46	3·39	3·66	−1·40	6
57 Peg	gM4	2·60	1·47	5·11	3·67	3·89	5·11	5·34	−2·86	2
71 Peg	gM5	3·17	1·62	5·32	3·65	3·84	5·00	5·23	−2·79	2
ρ Per	M4 II–III	3·34	1·65	3·35	3·51	3·68	4·75	4·92	−2·57	2
η Psc	G8 III	1·70	0·98	3·60	1·03	1·14	1·72	1·84	−0·23	6
ι Psc	F7 V	0·53	0·49	4·06	0·42	0·56	0·95	0·86	—	3
30 Psc	M3 III	3·34	1·62	4·37	3·27	3·45	4·57	4·77	−2·38	2
107 Psc	K1 V	1·36	0·84	5·26	0·71	—	1·54	1·48	−0·04	2
α Ser	K2 III	2·41	1·17	2·64	1·21	1·34	2·03	2·13	−0·38	2
λ Ser	G0 V	0·70	0·60	4·42	0·48	0·58	0·68	1·02	0·09	2
α Tau	K5 III	3·49	1·53	0·87	2·15	2·30	3·21	3·38	−1·22	20
γ Tau	K0 III	1·77	0·99	3·64	1·05	1·15	1·76	1·88	−0·25	4
δ Tau	K0 III	1·79	0·99	3·73	0·95	1·06	1·71	1·72	−0·18	2
ε Tau	K0 III	1·91	1·01	3·53	0·97	1·11	1·70	1·80	−0·20	2
o Tau	G8 III	1·44	0·86	3·61	1·12	1·10	1·72	1·72	−0·25	2
ξ U Ma	G0	0·58	0·57	3·77	0·45	0·59	1·02	1·01	0·06	2
ν Vir	M1 III	3·03	1·51	4·00	2·13	2·31	3·33	3·42	−1·26	3
ω Vir	gM6	—	1·62	5·33	3·67	3·84	4·91	5·15	−2·75	3
HR 483	G2 V	0·80	0·63	4·88	0·40	0·57	0·99	1·09	0·09	2
HR 36395	dM1	1·61	1·26	7·40	2·62	2·98	3·62	3·28	−1·56	1

Table 3. Observed spectral irradiance

An entry of 255–13 is to be read $0{\cdot}255 \times 10^{-13}$ W cm^{-2} μm^{-1}

name	sp. type	0·36	0·44	0·56	1·06	1·13	1·63	2·21
β And	M0 III	255–13	254–12	557–12	851–12	797–12	550–12	200–12
γ And	K3 II+A	415–13	246–12	439–12	410–12	364–12	221–12	814–13
λ And	G8 III–IV	229–13	917–13	199–12	654–13	610–13	328–13	117–13
R And	S4	395–15	342–14	794–14	236–13	232–13	183–13	773–14
β Aql	G8 IV	450–13	113–12	121–12	574–13	520–13	254–13	840–14
λ Aqr	MO	102–13	442–13	101–12	259–12	220–12	195–12	715–13
α Ari	K2 III	837–13	372–12	553–12	346–12	312–12	177–12	686–13
β Ari	A5 V	306–12	556–12	324–12	565–13	460–13	133–13	427–14
α Aur	G8 III+F	130–11	324–11	349–11	141–11	123–11	534–12	183–12
α Boo	K2 IIIp	471–12	246–11	396–11	267–11	251–11	148–11	541–12
η Boo	G0 IV	180–12	360–12	319–12	908–13	816–13	298–13	969–14
α C Ma	A1 V	173–10	276–10	243–10	230–11	181–11	515–12	169–12
α C Mi	F5 IV–V	200–11	344–11	264–11	691–12	608–12	208–12	756–13
α Cet	M2 III	144–13	155–12	361–12	704–12	652–12	494–12	196–12
κ Cet	G5 V	220–13	450–13	434–13	740–14	495–14	219–14	105–14
o Cet	M7e	170–14	499–14	108–13	566–12	105–11	848–12	445–12
τ Cet	G8 Vp	728–13	146–12	147–12	594–13	529–13	260–13	908–14
β Cnc	K4 III	854–14	701–13	141–12	147–12	139–12	932–13	344–13
RS Cnc	M6	199–14	108–13	262–13	518–12	523–12	430–12	198–12
β Com	G0 V	484–13	852–13	747–13	169–13	169–13	661–14	227–14
Y CVn	N3	—	370–14	269–13	132–12	999–13	128–12	679–13
ε Cyg	K0 III	799–13	292–12	389–12	211–12	188–12	105–12	374–13
γ Eri	M0 III	137–13	126–12	258–12	413–12	367–12	289–12	108–12
ε Eri	K2 V	344–13	978–13	118–12	566–13	476–13	235–13	904–14
16 Eri	gM3	989–14	722–13	154–12	508–12	462–12	355–12	157–12
β Gem	K0 III	271–12	986–12	128–11	628–12	568–12	274–12	975–13
η Gem	M3 III	125–13	876–13	197–12	508–12	455–12	334–12	140–12
κ Gem	G8 III	359–13	113–12	138–12	641–13	574–13	279–13	958–14
μ Gem	M3 III	100–13	110–12	274–12	704–12	695–12	506–12	226–12
51 Gem	M3	170–14	149–13	359–13	152–12	158–12	125–12	487–13
R Lep	N6e	—	—	176–14	366–13	370–13	584–13	492–13
α Lyr	A0 V	423–11	693–11	358–11	522–12	400–12	119–12	393–13
δ^2 Lyr	M4 II	382–14	292–13	701–13	406–12	355–12	339–12	126–12
11 L Mi	G8 IV–V	947–14	195–13	247–13	911–14	842–14	382–14	982–15
α Ori	M2 Iab	617–13	681–12	197–11	601–11	518–11	405–11	180–11
NL Ori	NB	425–16	235–14	111–13	499–13	424–13	452–13	190–13
o Ori	gM4	169–14	184–13	498–13	179–12	170–12	140–12	607–13
β Peg	M2 II–III	149–13	149–12	358–12	938–12	893–12	725–12	300–12
γ Peg	B2 IV	883–12	656–12	274–12	269–13	169–13	468–14	152–14
η Peg	G2 II–III	788–13	219–12	250–12	106–12	101–12	516–13	177–13
χ Peg	M2 III	214–14	207–13	457–13	764–13	718–13	496–13	203–13
55 Peg	M2 III	280–14	267–13	580–13	868–13	802–13	551–13	231–13
57 Peg	gM4	357–14	168–13	335–13	171–12	173–12	155–12	632–13
71 Peg	gM5	175–14	121–13	277–13	139–12	136–12	116–12	472–13
ρ Per	M4 II–III	913–14	719–13	170–12	748–12	724–12	562–12	216–12
η Psc	G8 III	328–13	106–12	135–12	607–13	550–13	274–13	100–13
ι Psc	F7 V	633–13	109–12	881–13	277–13	211–13	880–14	266–14
30 Psc	M3 III	356–14	289–13	662–13	234–12	228–12	185–12	737–13
107 Psc	K1 V	977–14	260–13	292–13	981–14	655–14	505–14	157–14
α Ser	K2 III	415–13	216–12	327–12	174–12	161–12	889–13	318–13
λ Ser	G0 V	392–13	717–13	637–13	172–13	155–13	499–14	224–14
α Tau	K5 III	783–13	791–12	167–11	211–11	199–11	134–11	519–12
γ Tau	K0 III	298–13	101–12	130–12	597–13	539–13	275–13	101–13
δ Tau	K0 III	268–13	933–13	119–12	497–13	454–13	241–13	803–14
ε Tau	K0 III	290–13	110–12	144–12	615–13	574–13	289–13	104–13
o Tau	G8 III	416–13	117–12	134–12	653–13	529–13	273–13	898–14
ξ U Ma	G0	797–13	133–12	116–12	306–13	286–13	124–13	403–14
ν Vir	M1 III	675–14	451–13	938–13	117–12	113–12	838–13	300–13
ω Vir	gM6	239–15	120–13	275–13	141–12	135–12	106–12	436–13
HR 483	G2 V	231–13	448–13	413–13	104–13	100–13	430–14	155–14
HR 36395	dM1	109–14	248–14	407–14	791–14	904–14	479–14	115–14

Goldberg & Pierce (1959), the published response curve of Johnson (1955), and the visual magnitude of the Sun as deduced by Martynov (1959) from the data of Stebbins & Kron (1957). A value for the spectral irradiance at the top of the atmosphere $H(\lambda)$ of

$$H(0{\cdot}56\,\mu\text{m}) = 3{\cdot}72 \times 10^{-12}\ \text{W cm}^{-2}\,\mu\text{m}^{-1}$$

was derived for a star of $V = 0{\cdot}0$ mag. This value, which is adopted here, is 2 % lower than that determined by Willstrop (1958) from direct comparison of the irradiance of a large number of stars to a standard lamp, and 5 % lower than the value recently published by Johnson (1965).

The accuracy of the absolute calibrations in the infrared region are difficult to estimate. Temperature fluctuations of the blackbody cavity probably represent one of the largest sources of error. An estimate of the magnitude of this effect was obtained by repeated cycling of the cavity over the temperature range from 400 to 500 °K. Copper-constantan thermocouples were placed at various points in the cavity and referenced to a 0 °C ice bath. The equilibrium temperature reached at the end of each cycle was recorded. The maximum variation observed was 1·1 °K. This temperature error amounts to a 4·9 % error in the radiance at 2·21 μm. The r.m.s. deviation of the temperature from the equilibrium value was 0·40 °K, which corresponds to a 1·8 % error in the radiance.

A check on the internal consistency of the calibrations can be made by comparing the relative spectral fluxes predicted by the calibration equations to known spectra. Table 4 shows such a comparison for an A0 V star. The values in the second column were obtained from the calibration equations with $W = X = Y = Z = V$. The values in the third column were obtained from graphical interpolation of the spectral distribution of emergent flux from Strom's (1964) model 5.

Table 4. A check on the absolute calibration

	$H(\lambda)/H(2{\cdot}21)$ (calib.)	$F(\lambda)/F(2{\cdot}21)$ (Strom)	% dev.
1·63	3·05	3·12	−2·2
1·13	10·43	10·81	−3·5
1·06	12·70	13·73	−7·5
0·55	100·0	93·20	+7·3

Errors in the measurement of the geometrical parameters pertinent to calculation of the blackbody flux will contribute to the over-all error; however, these are estimated to be less than 2 %. A greater unknown is the effect of aged and contaminated surfaces of the telescope optics. The reflexion from two freshly aluminized mirrors was included in laboratory calibrations of the photometer in an attempt to simulate the telescope optical path. It was not possible, however, to duplicate the actual condition of the telescope optics, nor was any attempt made to measure the infrared transmission of the telescope.

An estimate of the accidental errors of measurement including the errors in the correction for atmospheric extinction can be obtained from the reproducibility evidenced in a large number of observations of a single star. Measurements of this type obtained under a wide variety of atmospheric conditions indicate that the probable error of measurement is $0^m{\cdot}017$ in U–B, $0^m{\cdot}016$ in B–V and $0^m{\cdot}040$ in W, X, Y and Z.

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Consideration of the above remarks, together with the high degree of reproducibility of the stellar observations over a long period of time, lead one to the conclusion that the calibrations are good to within 10 % in the infrared region.

It is desirable to establish the relationship between the present infrared photometry and that obtained previously by other authors. Figure 1 relates $V\text{–}Z$ to the $V\text{–}K$ index of Johnson (1964) for 37 stars in common with his list. The best fit to the data gives the linear relation

$$(V\text{–}Z) = (V\text{–}K) - 0{\cdot}27, \tag{1}$$

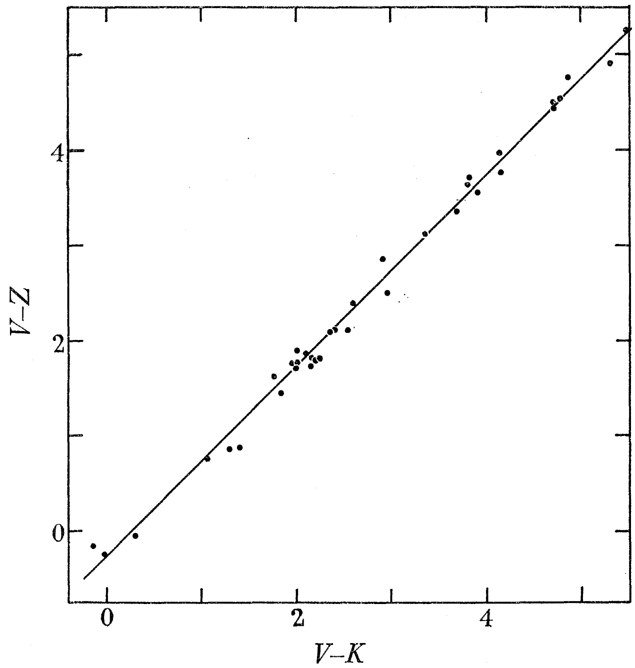

FIGURE 1. Comparison of the observed $V\text{–}Z$ index with $V\text{–}K$ of Johnson (1964) $(V\text{–}Z) = (V\text{–}K) - 0{\cdot}27$.

with a deviation from the mean of 0·08. The fit of the data over a range of six magnitudes is a measure of the linearity of both photometers. The difference in zero point of $-0^m{\cdot}27$ corresponds to a shift in the present photometry from $A0$ to AV. This may be seen by comparison of the colours of α Lyr with those of β Ari in table 2.

Figure 2 shows the relation between the magnitude m_x ($\lambda_e = 2{\cdot}20\,\mu$m) of Mitchell, Barnhart & Haynie (1961 *a*, *b*) and Z as determined by seventeen stars common to both lists. The equation of best fit is

$$Z = m_x + 0{\cdot}30, \tag{2}$$

which shows a zero point shift similar to (1).

Only seven stars of the present photometry are to be found in the list observed by Lunel (1960). These stars are plotted in figure 3 for the magnitudes

$$m_{A+\text{gel}}(\lambda_e = 1{\cdot}18\,\mu\text{m}) \quad \text{and} \quad m_{G+I}(\lambda_e = 2{\cdot}25\,\mu\text{m}).$$

The derived transformations are

$$W = m_{A+\text{gel}} + 0{\cdot}50, \tag{3}$$

and

$$Z = m_{G+I} + 0{\cdot}62. \tag{4}$$

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Table 5 shows a comparison between the infrared colours observed for α Tau, α Ori, μ Gem and o Cet with those calculated from the balloon spectra of Woolf, Schwarzschild & Rose (1964). Values for both their detectors A and B are given. The infrared indices were found by numerical integration of their published spectra weighted by the appropriate

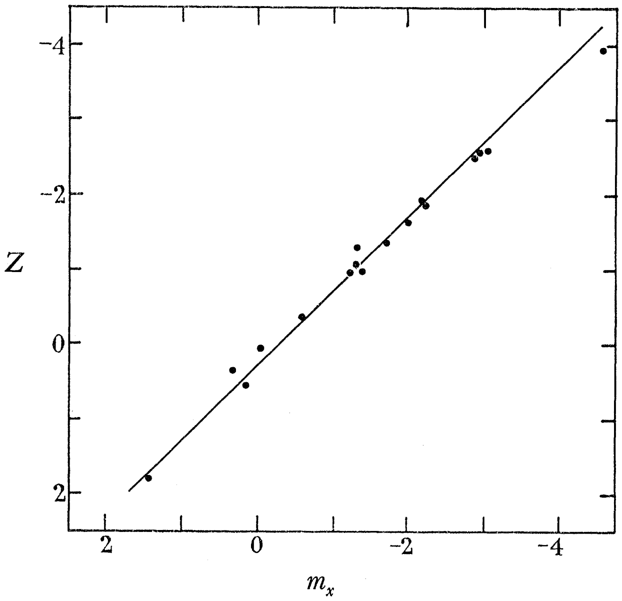

FIGURE 2. Comparison of m_x of Mitchell *et al.* (1961 *a*, *b*) with Z of the present photometry.

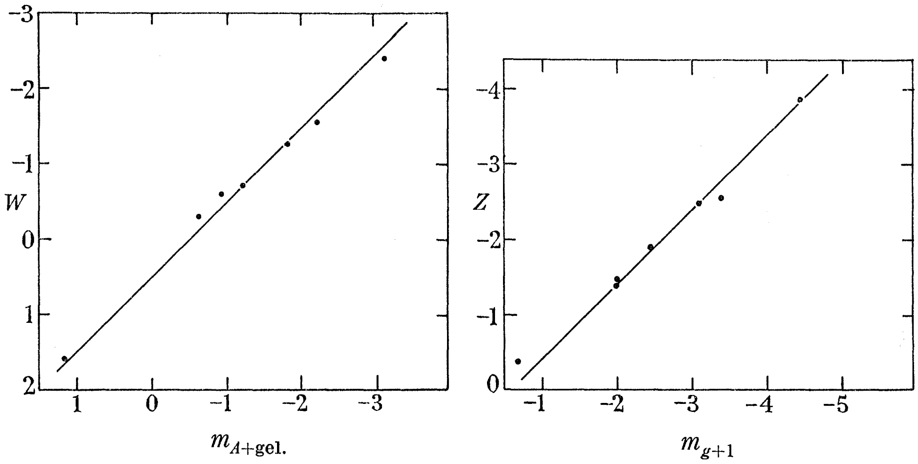

FIGURE 3. Comparison of the present photometry with that of Lunel (1960).

filter-band spectral response function, the zero point of the calculated colours being determined by their spectrum of α C Ma. The agreement between the observed and calculated colours, while not good is considered satisfactory. No systematic errors are indicated and the differences are of the same order as was evident when intercomparing the two detectors. The calculated colours of α Ori and o Cet could be too small due to the necessity to extrapolate their spectra to short wavelengths.

Table 5. Comparison of i.r. colours observed with those calculated from the spectra of Woolf *et al.* (1964)

Subscript *c* indicates colour calculated from the spectra of Woolf *et al.*; subscript *m*, measurements reported here.

star	$(W-Y)_c$	$(W-Y)_m$	$(W-C)_c$	$(W-Z)_m$
α Tau	1·09	1·06	1·37	1·23
	1·14		1·45	
α Ori	1·08	1·12	1·29	1·45
	1·04		1·23	
μ Gem	1·30	1·19	1·70	1·53
	1·11		1·34	
o Cet	1·70	1·99	2·31	2·50
	1·60		2·21	

Results

Bolometric corrections may be derived from the observations by integration of the spectral energy distributions defined by the monochromatic irradiance values given in table 3. The reduction equation as given by Walker (1964) is written

$$\text{b.c.}(*) = -2{\cdot}5 \log_{10} I(*) + 2{\cdot}5 \log_{10} I(\odot) + \text{b.c.}(\odot), \tag{5}$$

where the integral

$$I(*) = \frac{1}{H(1{\cdot}80)} \int_0^\infty H(n)\,\mathrm{d}n \tag{6}$$

is evaluated in wave-number units $n = 1/\lambda\,\mu\text{m}^{-1}$ for better convergence in the infrared region; $I(\odot) = I(*)$ evaluated for the Sun using the data of Goldberg & Pierce (1959); and b.c.$(\odot) = -0{\cdot}07$ the zero point of Kuiper's (1938) scale. The derived values of b.c. are presented in column 10 of table 2 and are plotted in figure 4 as a function of the colour index *V–Z*. These data, which are largely for stars of luminosity class III, show a well-defined curve with little scatter over a range of six magnitudes. The three stars that lie far from the mean curve are the *M* dwarf HR 36395 and the carbon stars R Leporis and Y Canum Venaticorum. Mean values for the bolometric correction as derived by Johnson (1964) for luminosity class III are also plotted in figure 4. His values have been adjusted by −0·07 mag in b.c. (to Kuiper's zero point) and −0·27 mag in *V–K* to correct for the zero point of the colour *V–Z*.

Figure 5 is a plot of the bolometric correction versus the *B–V* colour index. The curve is more poorly defined here with an indeterminancy in b.c. occurring at about *B–V* = 1·65 Once again, the *M* dwarf and the carbon stars are well separated from the general curve as is the super-giant α Orionis, M2 Iab.

The spectral type–bolometric correction relation is given in table 6. The data presented here are means of the quantities given in table 2. Since the number of stars is small the means may not be highly significant and thus the relation in table 6 is poorly defined. Comparison with the data of Walker & D'Agati (1964) shows the new values to be somewhat larger for spectral types later than K4 III. This increase is attributed to the new photometry at 1·63 μm. Comparison with Johnson (1964), after correction for zero point, shows good agreement.

It is well known that for stars of solar type and later the main source of opacity is

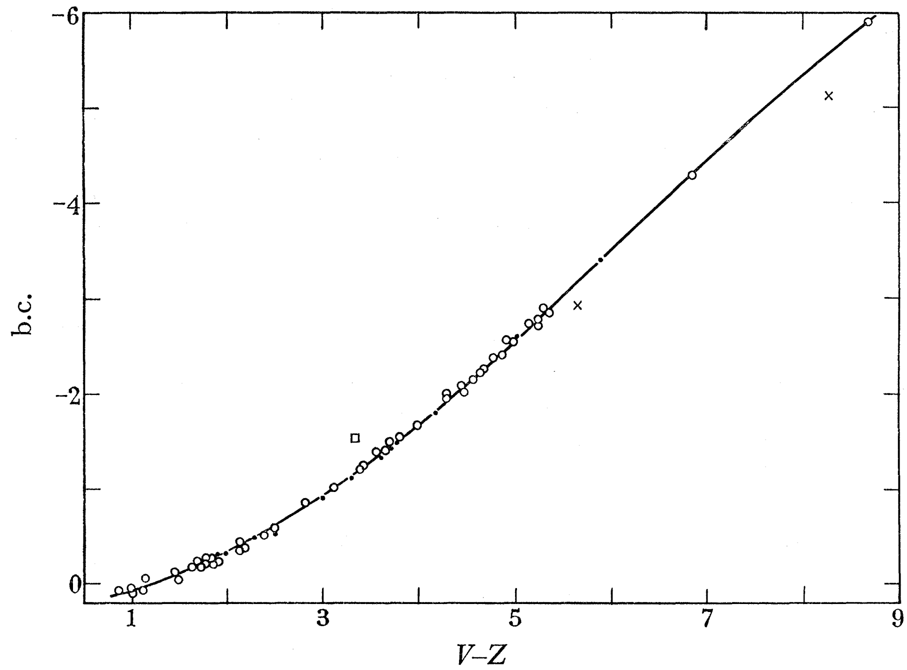

FIGURE 4. Relation between the bolometric correction and the infrared index V–Z.
●, Johnson (see text); ○, present data; □, M dwarf; ×, carbon stars.

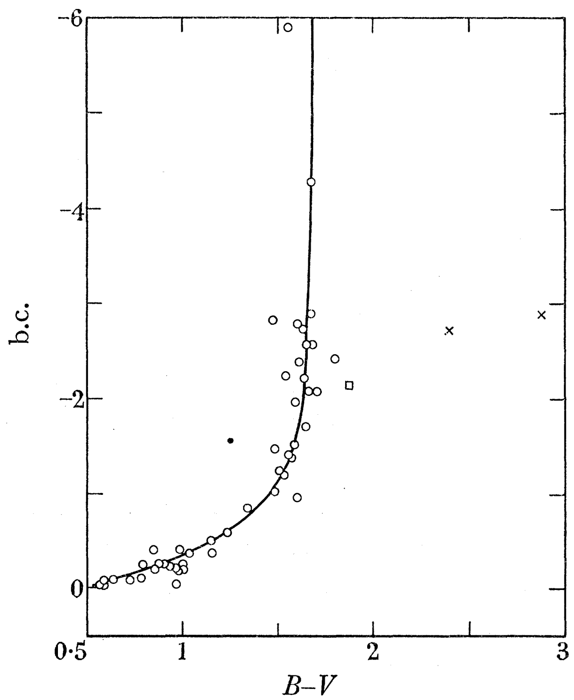

FIGURE 5. Relation between the bolometric correction and the colour B–V.
●, M dwarf; ×, carbon stars; □, supergiant.

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probably due to the negative hydrogen ion. The absorption coefficient of H^- exhibits a pronounced minimum at 1·65 μm; thus one may expect the stellar flux distribution to show a maximum in this region. This maximum is evident to a small degree in the solar spectrum, is a pronounced feature in the spectra of Aldebaran obtained by Woolf *et al.* (1964) with their balloon-borne telescope, and becomes the dominant spectral feature in the model atmospheres developed for cool stars by Gingerich & Kumar (1964). Observations of late-type stars in the 1·65 μm region can thus be expected to provide information that bears directly on the problem of the opacity of late stellar atmospheres.

Table 6. The bolometric correction for stars of late spectral type

sprectrum	b.c.	spectrum	b.c.	spectrum	b.c.
G2 III	−0·22	M0 III	−1·44	G2 V	−0·07
G8 III	−0·25	M2 III	−1·54	G8 V	−0·12
K0 III	−0·26	M3 III	−2·12	G8 IV	−0·27
K2 III	−0·50	M4 III	−2·42	K2 V	−0·26
K4 III	−1·03	MB	−2·80	M2 II	−2·07
K5 III	−1·22	M6	−4·30	M4 II	−2·57

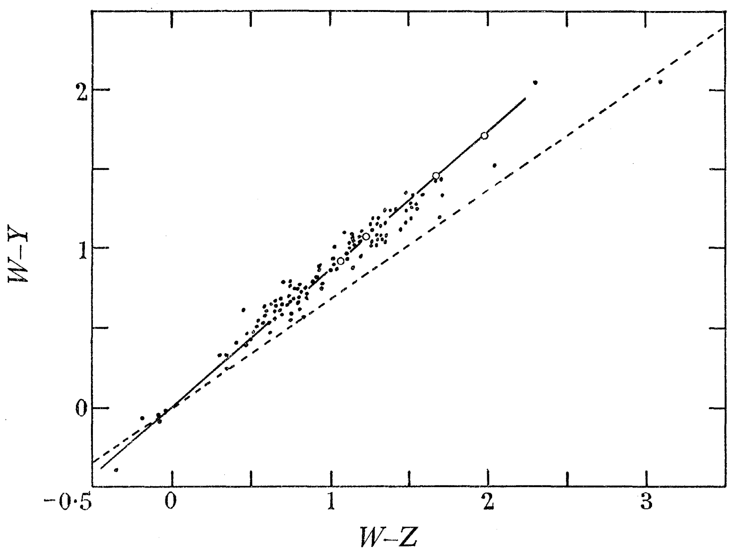

Figure 6. Infrared colour-colour diagram about the wavelength 1·63 μm. —o—, Gingerich models; – – –, blackbody colours.

Figure 6 is an infrared colour–colour diagram relating observations at effective wavelengths of 1·06, 1·63 and 2·21 μm. The dotted line is the locus of calculated colours for blackbody radiation. The stellar observations lie clearly above this line, indicating an excess of radiation in the *Y* band. That this is in agreement with theoretical predictions can be seen from theoretically derived colours also plotted. These colours were obtained from numerical integration of the emergent flux distributions of four model atmospheres of O. Gingerich (1964), private communication, values of the flux being weighted by the filter-band spectral response. The Gingerich models are for effective temperatures T_e = 4250, 4000, 3500 and 3200 °K with $\log_{10} g$ = 3·0 and include the effects of Rayleigh scattering from molecular and atomic hydrogen. The model atmosphere for an A0 V star due to Strom (1964) was used to determine the zero point of the calculated infrared colours. The clustering of the observations along the line defined by the model calculations is interpreted as observational support of the model predictions.

Measurements of absolute stellar irradiance when combined with results of model atmosphere calculations offer a simple photometric technique for estimating stellar diameters. Consider a flux of πF_λ at the surface of a star of radius R. If, following Irwin (1962), we relate the intensity J at a point on the stellar disk characterized by a zenith angle γ to the intensity J_c at the centre by the equation

$$J = J_c(1 - x + x\cos\gamma), \tag{7}$$

then the irradiance H_λ observed at a distance d from the star will be given by

$$H_\lambda = \tfrac{1}{4}\pi\,\alpha^2 F_\lambda(1 - \tfrac{1}{3}x). \tag{8}$$

The quantity $\alpha = 2R/d$ is the angular diameter of the star, and x is the coefficient of limb darkening.

To the degree of approximation implied by (7), equation (8) may be solved for α by using H_λ from table 3 and F_λ and x from the model atmosphere. The models employed here are those due to Gingerich (1964), private communication, referred to above, and include calculations of limb darkening. His calculations were fit to equation (7) and the darkening coefficients determined.

Application of the method has been restricted to the infrared at 2·21 μm for a number of reasons. First, in order to apply the method, one must assume an effective temperature for the star. This was done by using the spectral classification of the star and the spectral type–temperature relationship of Harris (1963). Such a procedure will result in a temperature with an uncertainty on the order of a few hundred degrees. This uncertainty will introduce an error into α through the calculated flux F_λ. The magnitude of this effect can be seen in table 7, where the percentage error introduced into α due to an uncertainty of 100 °K is given for several effective temperatures and wavelengths. As can be readily seen, the solution at 2·21 μm is one-third to one-quarter as sensitive to temperature variations as that in the visual.

Table 7. The percentage error in the derived angular diameter due to an error of 100 °K in the assumed temperature

T_e	0·36 μm	0·44 μm	0·56 μm	2·21 μm
5200	7·1	5·9	4·7	1·6
4200	10·8	8·9	7·1	2·3
3200	18·9	15·5	12·3	3·2

Secondly, present model atmospheres for late-type stars do not take into account the additional opacity in the short wavelength region due to molecular bands. It is well known that in the early M stars the great number and strength of the molecular bands have depressed the continuum far below its expected value (Aller 1960). Use of existing model atmospheres to predict the flux of radiation at wavelengths in the ultraviolet, blue or

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visible can therefore be expected to lead to large errors in F_λ. The situation is considerably improved in the infrared at 2·21 μm, at least for stars earlier than about M4 where absorption by the H_2O molecule first becomes important. The principal molecular absorber in this region is the $\Delta v = 2$ vibration-rotation band of CO. This band absorbs at wavelengths beyond its band head at 2·30 μm (McCammon, Munch & Neugebauer 1967) and thus has little effect on measurements in the Z filter band. Indeed, the influence of C0 absorptions should be negligible for stars earlier than about M0.

Consideration of the expected degree of limb darkening for late-type stars provides a third reason for choosing the infrared region for making photometric measures of stellar diameters. The limb darkening calculations of Gingerich (1964), private communication, show that for the range of temperatures considered here, the limb darkening term of equation (8) can amount to 20 % of α in the visible portion of the spectrum while typical values at 2·21 μm lie in the range of 5 to 12 %.

Finally, observation of the stellar irradiance is affected by extinction in the Earth's atmosphere. Table 8 shows two extremes of extinction observed at the Agassiz Station of Harvard Observatory (Walker 1966). It is clear that the extinction at Z is on the order of a factor of two less than at V. The situation becomes even more favourable in the infrared at a dry, high altitude observatory; since, as may be seen from the tabulated values of the aerosol component of extinction $k_{Z,a}$, a large part of the extinction at Z is due to Atmospheric water vapour.

Table 8. Extremes of extinction observed at Agassiz Station, Harvard, Massachusetts

The dimensions of k are magnitudes per unit air mass. The final column gives the water-vapour content in precipitable cm.

date	k_U	k_B	k_V	k_Z	$k_{Z,a}$	H_2O
11 Mar. 65	1·55	0·92	0·77	0·40	0·31	0·44
2 Dec. 64	0·85	0·38	0·23	0·11	0·05	0·24

Angular diameters calculated from (8) and based upon observations at 2·21 μm are given in table 9 for stars later than G5 III. An indication of the accuracy of the method can be had by comparing the radiometric diameters α with those measured by interferometric techniques. Table 10 shows this comparison for six stars with measured angular diameters. The auxiliary quantity α_e is the radiometric diameter which would be derived for an undarkened star ($x = 0$). The calculation for α C Ma was based upon Strom's (1964) model with no correction made for darkening.

Knowledge of the angular subtent of the star, its bolometric correction and V magnitude is all that is required to determine its effective temperature T_e. From the definition of T_e,

$$\sigma T_e^4(*) = \left(\frac{1}{\alpha}\right)^2 \int_0^\infty H_\lambda \,d\lambda, \tag{9}$$

we may write relative to the Sun, $\odot$,

$$\log_{10}\frac{T_e(*)}{T_e(\odot)} = -\tfrac{1}{2}\log_{10}\frac{\alpha(*)}{\alpha(\odot)} - 0{\cdot}1 m_{\text{bol.}}(*) + 0{\cdot}1 m_{\text{bol.}}(\odot), \tag{10}$$

Table 9. Derived values of stellar radius and effective temperature

star	sp. type		a	R/R_e	T_e
β And	M0	III	0″014	34	3810
γ And	K3	II+A	0·0082	172	4160
λ And	G8	III–IV	0·0027	6·6	4720
R And	S4		0·0028	—	3550
β Aql	G8	IV	0·0023	3·5	4430
λ Aqr	M0		0·0083	73	3710
α Ari	K2	III	0·0074	18	4300
α Aur	G8	III+F	0·011	16	4960
α Boo	K2	IIIp	0·021	25	4030
α Cet	M2	III	0·014	(490)	3680
o Cet	M	7e	0·057	460	1960
β Cnc	K4	III	0·0054	41	4010
RS Cnc	M6		0·021	—	2830
Y CVn	N3		0·018	—	2260
ϵ Cyg	K0	III	0·0050	12	4640
γ Eri	M0	III	0·010	—	3890
16 Eri	gM3		0·013	81	3510
β Gem	K0	III	0·0081	9·2	4790
η Gem	M3	III	0·013	105	3480
κ Gem	G8	III	0·024	10	5010
μ Gem	M3	III	0·016	80	3470
51 Gem	M4	III	0·0098	115	3000
R Lep	N6e		0·015	—	2080
δ^2 Lyr	M4	II	0·010	(520)	3800
α Ori	M2	Iab	0·042	885	3420
BL Ori	NB		0″0054	—	3130
o Ori	gM4		0·010	210	3120
β Peg	M2	II–III	0·019	130	3410
χ Peg	M2	III	0·0046	69	3670
55 Peg	M2	III	0·0049	47	3700
57 Peg	gM4		0·011	(380)	2980
71 Peg	gM5		0·010	—	2940
ρ Per	M4	II–III	0·021	270	3030
η Psc	G8	III	0·0024	36	4940
30 Psc	M3	IV	0·0092	23	3470
α Ser	K2	III	0·0050	11	4430
α Tau	K5	III	0·022	48	3820
γ Tau	K0	III	0·0026	—	4720
δ Tau	K0	III	0·0023	15	4830
ϵ Tau	K0	III	0·0027	16	4740
o Tau	G8	III	0·0023	22	5040
ν Vir	M1	III	0·0054	44	3890
ω Vir	gM6		0·0067	71	2940

Table 10. Comparison of derived angular diameters with interferometer values α_i as tabulated by Johnson (1964)

star	α_i	α_e	α
α Tau	0·020	0·020	0·022
α Boo	0·020	0·019	0·021
α Ori	0·041	0·038	0·042
β Peg	0·021	0·017	0·019
o Cet	0·053	0·052	0·057
α C Ma	0·0072	0·0077	—

which after substitution of the following quantities, $T_e(\odot) = 5778$ °K, $m_{\text{bol.}}(\odot) = -26{\cdot}87$, $\alpha(\odot) = 1958{\cdot}7$ sec arc and $m_{\text{bol.}} = V + \text{b.c.}$, yields

$$\log_{10} T_e(*) = -\tfrac{1}{2}\log_{10}\alpha(*) - 0{\cdot}1(V+\text{b.c.}) + 2{\cdot}721. \tag{11}$$

A comparison of the effective temperatures derived from equation (11) and the data from table 2 with fundamental temperatures quoted by Harris, Strand & Worley (1963) is given in table 11. The agreement for α Boo and α Tau is considered satisfactory. The known variability of o Cet's temperature makes a comparison difficult. The observations were obtained just past minimum; therefore, temperatures on the low side seem reasonable. Both α Orionis and β Pegasi are affected by interstellar reddening. The agreement appears good for α Ori; however, the β Peg value of 3410 °K is more in line with a recent determination of 3300 °K by Johnson (1964).

Table 11. Comparison of derived temperatures

star	T_e (present study) without darkening	T_e (present study) with darkening	T_e (Harris)
α Ori	3780	3420	3460
α Boo	4460	4030	4090
α Tau	4040	3820	3780
β Peg	3620	3410	3080
o Cet	2080	1960	2360

Table 9 summarizes the results of applying (8) and (11) to the data of tables 2 and 3 for all stars later than G5 III. Values tabulated for the stellar radii relative to the solar radius R/R_0 are based upon parallax data obtained from the Yale Bright Star Catalogue. Radii enclosed in parentheses are for stars with parallax less than 0·005. The derived mean spectral type–temperature relationship is presented in table 12 where it may be compared with Harris *et al.* (1963) 'revised scale' and the recent determination of Johnson (1964). The variation of effective temperature with the infrared index W–Z is shown in figure 7. The point at W–Z = 3 is the variable R Leporis. It appears abnormally red both here and in the b.c. V–Z relationship (see table 2). The best straight line fit to the data, omitting R Lep, is given by

$$\log_{10} T_e = 3{\cdot}854 - 0{\cdot}221(W-Z). \tag{12}$$

If extrapolated to higher temperatures, this line passes close to the point $\odot$ plotted for the effective temperature of the Sun and $(W-Z)_\odot$ determined for a star of 'mean solar type'. The dashed curve is the locus of W–Z colours for a blackbody at temperature $T_b = T_e$. The observed colours lie in general to the red of the blackbody curve, corresponding very roughly to an infrared colour temperature $T_c = 0{\cdot}89T_e$.

The stellar temperatures derived above depend critically upon the accuracy of the bolometric corrections and angular diameters determined from the observations. To derive bolometric corrections from integration of the stellar spectral energy distribution, one must interpolate the spectrum over relatively wide regions in the infrared where observation is not possible owing to absorption by atmospheric water vapour and carbon

TABLE 12. THE RELATION OF EFFECTIVE TEMPERATURE TO SPECTRAL TYPE FOR STARS OF LUMINOSITY CLASS III

Sp.	Johnson	Harris	Walker
G8	4850	4980	4988
K0	4760	4720	4744
K2	4460	4280	4365
K3	4340	3950	4160
K4	3960	3730	4010
K5	3820	3500	3820
M0	3680	—	3803
M2	3600	—	3683
M3	3370	—	3473
M4	3060	—	3015

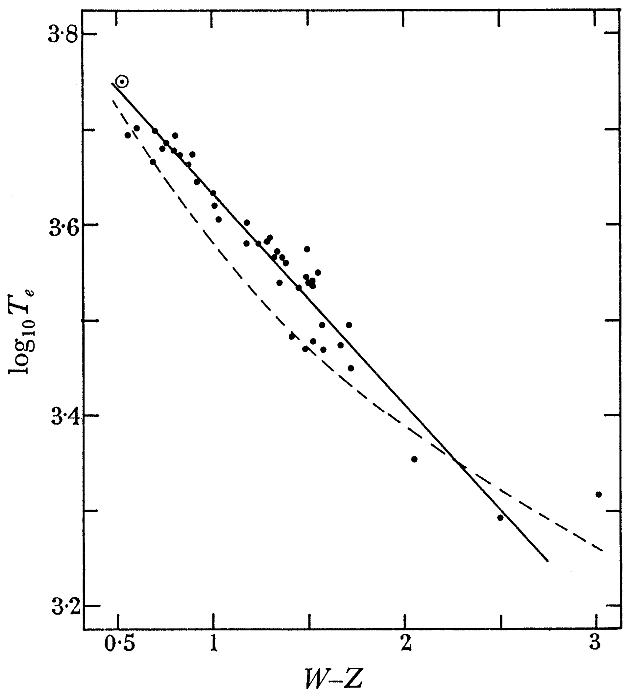

FIGURE 7. Empirical relation between effective temperature and the infrared index W–Z. The point ⊙ is the effective temperature of the Sun plotted for a star of mean solar colour. ——, $\text{Log}_{10} T_e = 3.854 - 0.221\ (W\text{–}Z)$; – – –, blackbody locus, $T_e = T_b$.

dioxide. Such a technique will not be subject to large errors if the stellar spectrum is a smooth function over the interval in question. This appears to be the case for giants earlier than about M4, where molecular absorptions due to stellar water vapour first become observable (see Woolf *et al.* 1964; Spinrad & Newburn 1965). The strong vibration-rotation bands of H_2O at 1·37 and 1·87 μm rapidly increase in strength with late spectral type. These strong absorption features, which are greatly broadened at stellar temperatures, coincide in wavelength to the Earth's atmospheric absorption bands; therefore, interpolation across these bands will lead to an overestimate of the bolometric correction for late spectral types. An extreme example is *o* Cet. The balloon spectra of Woolf *et al.*

(1964) show that the water-vapour absorptions remove approximately 30% of the total emergent flux. Taking this reduction of flux into account reduces the bolometric correction by 0·39 mag and decreases the effective temperature by 9·2% (180 °K).

The angular diameters are sensitive to the absolute calibration of the photometer and the absolute value of the emergent flux at the surface of the star calculated from the models; a 10% error in either quantity resulting in a 5% error in the derived radius. Recent models for late-type stars constructed by Gingerich, Latham & Linsky (1966) show that although the flux at 2·21 μm is relatively insensitive to the temperature and composition of the model, it is sensitive to the luminosity class of the star. Their data indicate a 40% increase in the 2·21 μm flux for a change in surface gravity from $\log_{10} g = 3$ to $\log_{10} g = 1$ at constant $T_e = 2500$ °K. Inclusion of water-vapour absorption in these models does not appreciably alter the flux at 2·21 μm (Gingerich & Linsky 1966, private communication).

The temperatures given in table 9 are for luminosity class III and spectral types M4 and earlier. Thus, they are free of the effects of strong molecular absorptions in the infrared as well as the effects of low surface gravity. This same statement applies to most of the data in table 8; however, stars later than M4 and of luminosity class different from III can be expected to have greater uncertainties in the derived temperatures and angular diameters. The values for F_λ used in the calculation were taken from atmospheres with $\log_{10} g = 3$. If, according to Aller (1960), the surface gravity of a giant star decreases with decreasing temperature, then the angular diameters derived here will be systematically too large at the later spectral types.

The primary reason for the photometry at 1·13 μm was to attempt a photometric determination of water-vapour in the atmospheres of late-type stars. Twelve stars with spectral type later than M4 III were observed specifically for this purpose. Six stars were in the lists observed by Spinrad & Newburn (1965) and Spinrad, Pyper, Newburn & Younken (1966); and four were observed by Woolf *et al.* (1964) to show measurable water-vapour absorptions. The present photometry could detect no reduction of intensity in the X filter band attributable to stellar water-vapour absorption. This was due, in part, to the increased uncertainties introduced into the measurements by extinction corrections for atmospheric water vapour; and, in a larger part, to the loss in sensitivity of the measurement due to an excessive spectral bandwidth passed by the X band filter. The carbon stars, Y C Vn and Bl Ori, did show a marked reduction of intensity in the X band. This is attributed to absorption by the 1·1 μm band of the CN molecule. Spectra of these stars taken in the photographic infrared by McKellar (1954) clearly show the CN bands beginning at about 1·09 μm and increasing in intensity toward longer wavelengths. These observations support the suggestion of Solomon & Stein (1966) that the peculiar energy distributions for the carbon stars observed by Johnson, Mendosa & Wesniewski (1965) are due in part to absorptions by CN and C_2. Indeed the C_2 band at 1·21 μm would have a strong effect on the response in the X filter band.

I wish to express my appreciation to Dr Carl Sagan for his many helpful suggestions as well as for his sustained interest in this research. I also extend my gratitude to Dr Owen Gingerich who generously supplied me with model atmospheres for late-type stars and who, along with Jeffrey Linsky, critically read an early version of the manuscript. This research was partially supported by the Air Force In-House Laboratory Independent Research Fund.

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